UniODA vs. Bowker's Test for Symmetry: Region of Residence and Time

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Bowker’s test for symmetry is a generalization of McNemar's test for correlated proportions that is used for tables having more than two categories. The present study contrasts results achieved using Bowker’s test versus an iterative UniODA-based procedure.

Bowker’s test for symmetry—which is identical to McNemar’s test for correlated proportions for 2x2 tables—is used with square tables having more than two categories. For both of these tests the null hypothesis is that the cell proportions are symmetric: that is, $p_{ij} = p_{ji}$ for all pairs of table cells. Both tests are inherently two-tailed (i.e., the alternative hypothesis is non-directional), and Bowker’s test is chi-square asymptotic-based: it ignores empty cell pairs, and the minimum expectation assumption must be satisfied for all cells in the table (however, an alternative exact methodology has been developed).

The present exposition compares results achieved by using Bowker’s test versus an iterative UniODA-based procedure previously demonstrated in analysis of turnover tables reflecting serial voting behavior, and in analysis of Markov transition models of geological soil sections. The current application models both stability as well as change in region of residence (North East, Midwest, South, West) using data collected by the US Bureau of the Census in 1980 and 1985. These data were analyzed by Bowker’s method using log-linear models that tested for independence, quasi independence, and quasi symmetry—all of which failed to achieve satisfactory fit and were thus found to be statistically untenable.

Table 1: Data for Example

<table>
<thead>
<tr>
<th>Region of Residence</th>
<th>In 1980</th>
<th>North East</th>
<th>Midwest</th>
<th>East</th>
<th>South</th>
</tr>
</thead>
<tbody>
<tr>
<td>In 1985</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>North East</td>
<td>11,607</td>
<td>100</td>
<td>366</td>
<td>124</td>
<td></td>
</tr>
<tr>
<td>Midwest</td>
<td>87</td>
<td>13,677</td>
<td>515</td>
<td>302</td>
<td></td>
</tr>
<tr>
<td>East</td>
<td>172</td>
<td>225</td>
<td>17,819</td>
<td>270</td>
<td></td>
</tr>
<tr>
<td>South</td>
<td>63</td>
<td>176</td>
<td>286</td>
<td>10,192</td>
<td></td>
</tr>
</tbody>
</table>

The first step of the UniODA analysis tested the *a priori* hypothesis that region of residence was stable between 1980 (treated as the attribute) and 1985 (treated as the class variable). This stability model specified that the
A fourth UniODA statistical analysis is not conducted: for a categorical design with $C$ class categories and $C$ non-empty cells, classification accuracy and ESS will always be perfect. A final exploratory model can however be identified without ascertaining Type I error or the ESS statistic: as seen in the UniODA data table, all cells already successfully modeled were set to zero. The resulting model identified the movement of 87 residences from the Midwest to the Northeast, versus the comparable 1.1-fold greater movement of 100 residences from the North East to the Midwest; as well as the movement of 270 residences from the East to the South, versus the comparable 1.1-fold greater movement of 286 residences from the South to the East.

Together the four UniODA models correctly classified all of the data in the original table. The three UniODA models for which statistical significance was ascertained together correctly classified 55,238 (98.7%) of the total of 55,981 observations in the table, yielding an overall ESS statistic of 98.2. The initial $a$ priori analysis showed that the overwhelming effect was that region of residence was stable between 1980 and 1985. Nevertheless two exploratory analyses identified eight specific statistically reliable instances of marginal dissymmetry. The final UniODA model revealed that the four residual cells in the table reflected marginal symmetry vis-à-vis comparable proportional changes. All of the analyses reported herein together required a total of 7 CPU seconds to solve running UniODA software on a 3 GHz Intel Pentium D microcomputer.

References


9. The UniODA syntax used to conduct this analysis is:

```
OPEN DATA;
OUTPUT EXAMPLE.OUT;
CATEGORICAL ON;
TABLE 4;
CLASS COL;
DIRECTIONAL < 1 2 3 4;
MCARLO ITER 25000 TARGET .001 STOP 99.999;
DATA:
11607 100 366 124
87 0 515 302
172 225 0 270
63 176 286 0
END DATA;
GO;
```

10. The UniODA syntax used to conduct this analysis is:

```
OPEN DATA;
OUTPUT EXAMPLE.OUT;
CATEGORICAL ON;
TABLE 4;
CLASS COL;
MCARLO ITER 25000 TARGET .001 STOP 99.999;
DATA:
0 100 366 124
87 0 515 302
172 225 0 270
63 176 286 0
END DATA;
GO;
```

11. The same UniODA syntax used to conduct the prior analysis was used for the present analysis, but with the following data table:

```
0 100 0 124
87 0 515 0
0 225 0 270
63 0 286 0
```

12. The same UniODA syntax used to conduct the prior analysis was used for the final analysis, but the MCARLO command was eliminated and the following data table was used:

```
0 100 0 0
87 0 0 0
0 0 0 270
0 0 286 0
```

**Author Notes**

This study involved secondary data analysis of published de-identified data and was exempt from Institutional Review Board review.

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